

# STUDY OF DIFFERENCES BETWEEN REAL AND EFFECTIVE HEAT TRANSFER COEFFICIENTS TO PROVIDE CORRECT DATA ON TEMPERATURE FIELD CALCULATIONS AND COMPUTER SIMULATIONS DURING HARDENING OF STEEL

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## Abstract

The paper analyses contemporary methods and probes for testing liquid media used as a quenchant in heat treating industry. It is shown that lumped-heat-capacity method, often used for testing liquid media, produces big errors during transient nucleate boiling processes due to incorrect calculation condition  $Bi \leq 0.25$  caused by use effective heat transfer coefficient (HTC). The effective heat transfer coefficients (HTCs), utilized for this purpose, are almost seven times less as compared with real HTCs that results in incorrect calculation the value of Bi. Instead of lumped-heat -capacity method, a general cooling rate equation is proposed for HTC calculation. It is underlined that effective HTCs can be used only for approximate core cooling rate and core cooling time of steel parts calculations. For investigation cooling capacity of liquid quenchants, including initial heat flux densities, HTCs and critical heat flux densities, high developed technique of solving inverse problem should be used based on accurate experimental data generated by testing liquid media with the Liscic/Petrofer probe or other similar technique.

**Keywords:** lumped-heat-capacity method, incorrectness, effective HTC, real HTC, differences, probes, inverse problem.

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## 1. Introduction

In this paper effective heat transfer coefficients (HTCs) and effective Kondrat'ev numbers  $Kn_{ef}$  are considered and is shown possibility of their use for core cooling time and core cooling rate calculations of different steel parts [1–3]. A difference between effective and real HTCs is discussed in the paper. Such consideration is very important because historically in heat treating industry mainly effective HTCs are used for development recipes of cooling. It is impossible to derive real HTC during transient nucleate boiling process from testing standard probe with one thermocouple at the core since nucleate boiling process always creates temperature gradient through the section of the probe and in this case surface temperature of the probe differs significantly from its core temperature. It means that the lumped-heat-capacity method [4–7] doesn't work during transient nucleate boiling process. Instead of the lumped-heat-capacity method, a universal cooling rate correlation of regular condition theory is used for evaluating effective Kondrat'ev numbers and HTCs. Real HTCs require exact measurement surface temperature of the probe during transient nucleate boiling process. That was possible to do using French or Liscic/Nanmac probes [8, 9]. Currently it can be calculated by solving inverse problem based on experimental data provided by Liscic/Petrofer probe [10]. This important problem is discussed below.

## 2. The main differences between real and effective heat transfer coefficients

In this paragraph a difference between effective and real heat transfer coefficients (HTCs) is discussed. As known, the heat transfer coefficient during transient nucleate boiling process is evaluated as a ratio of the heat flux density produced by bubbles to the overheat of the boundary layer [11, 12], i. e.

$$\alpha_{nb} = \frac{q}{T_{sf} - T_s}. \quad (1)$$

Here  $\alpha_{nb}$  is real HTC during transient nucleate boiling process;  $\alpha_{ef}$  is effective HTC;  $q$  is heat flux density;  $T_{sf}$  is surface temperature;  $T_s$  is saturation temperature;  $T_m$  is bath temperature.

In heat treating industry, historically, the heat transfer coefficient (HTC) during transient nucleate boiling process is evaluated as:

$$\alpha_{ef} = \frac{q}{T_{sf} - T_m}, \quad T_m \ll T_s. \quad (2)$$

That creates misunderstanding and leads to rather big errors during temperature fields calculations within the transient nucleate boiling processes due to big difference between real and effective HTCs (**Table 1**) [12, 13]. When calculating data shown in **Table 1**, it was assumed that overheat at the beginning of transient nucleate boiling process was equal to 14 °C. As seen from **Table 1**, the ratio  $\alpha_{real}/\alpha_{eff}$  depends on temperature of a liquid.

**Table 1**

Ratio of real HTC to effective HTC versus temperature of water

Temperature, °C	15	20	30	40	60	80	90	95
$\alpha_{real}/\alpha_{eff}$	7.07	6.71	6.00	5.29	3.86	2.43	1.71	1.26
$\alpha_{real}/\alpha_{eff}, \%$	707	671	600	529	386	243	171	126

One must know that HTC is related to the difference  $\Delta T_s = T_{sf} - T_s$ , not to difference  $\Delta T_m = T_{sf} - T_m$ . The critical radius of a bubble growth depends on overheat of a boundary layer  $\Delta T_s = T_{sf} - T_s$  which is determined as [11]:

$$R_{cr} = \frac{2\sigma T_s}{r^* \rho \Delta T_s}, \quad (3)$$

where  $R_{cr}$  is a critical radius of a bubble which is capable to grow and function;  $\sigma$  is a surface tension (N/m);  $T_s$  is a saturation (boiling) temperature (K);  $r^*$  is latent heat of evaporation (J/kg);  $\rho$  is vapor density (kg/m<sup>3</sup>); and  $\Delta T_s = T_{sf} - T_s$  is wall overheat.

According to author [11], the average heat flux density  $q$  during nucleate boiling is proportional to the cube of temperature difference  $\Delta T_s = T_{sf} - T_s$ :

$$q \approx \Delta T_s^3, \quad (4)$$

that is why overheat  $\Delta T_s = T_{sf} - T_s$  is small from the very beginning of cooling.

Tolubinsky's equation (5) for calculating HTCs during transient nucleate boiling process one can find in [11]

$$\frac{\alpha}{\lambda} \sqrt{\frac{\sigma}{g(\rho - \rho^*)}} = 75 \cdot \left( \frac{q}{r^* \rho^* W^*} \right)^{0.7} \cdot \left( \frac{v}{a} \right)^{-0.2}. \quad (5)$$

Here  $\alpha$  is real HTC during nucleate boiling process in W/m<sup>2</sup>K;  $\lambda$  is thermal conductivity of liquid in W/mK;  $g$  is gravitational acceleration in m/s<sup>2</sup>;  $\rho$  is liquid density in kg/m<sup>3</sup>;  $q$  is heat flux density in W/m<sup>2</sup>;  $W^*$  is vapor bubble growth rate in m/s;  $\nu$  is kinematic viscosity in m<sup>2</sup>/s;  $a$  is thermal diffusivity of liquid in m<sup>2</sup>/s.

Real HTCs during nucleate boiling process vs. temperature of water salt solution are presented in **Table 2** [2]. **Table 2** shows the real heat transfer coefficients in W/m<sup>2</sup>K which are responsible for developing temperature gradients during quenching of probes and steel parts. Calculations of HTCs were made for maximal critical heat flux density of water salt solutions which was equal to 15 MW/m<sup>2</sup> [2]. Dimensionless correlations of authors [11, 14] were used for this purpose.

**Table 2**

Real HTC's in W/m<sup>2</sup>K during nucleate boiling process depending on the temperature of water solution when heat flux density is 15 MW/m<sup>2</sup>

Temperature of water solution	Tolubinsky	Shekrladze	Average
10	152248	176546	164397
20	193929	243641	218785
40	224989	241615	233302
60	271273	271323	271298

As seen from **Table 2**, real HTC's are very large when heat flux density approaches the critical value 15 MW/m<sup>2</sup>. It means that in the Inconel 600 standard probe 12.5 mm in diameter maximal temperature gradient will arise because Biot number Bi is approximately 68.

### 3. Lumped-heat-capacity method

In FSU was used standard silver spherical probe 20 mm in diameter with one thermocouple instrumented at the center of the probe. Many researchers used that standard probe for evaluating HTC's [15]. They explored well known approach which states that core temperature is almost equal to surface temperature if Biot number  $Bi \leq 0.25$ . Since effective HTC for brine is within 8000 W/m<sup>2</sup>K – 10000 W/m<sup>2</sup>K, the Biot number is:

$$Bi = \frac{10,000 \text{ W/m}^2\text{K}}{400 \text{ W/mK}} \times 0.01 \text{ m} = 0.25.$$

Based on this data, all believed that core temperature and surface temperature were almost equal to each other and in many books and handbooks were provided “accurate” data for engineers which are shown in **Fig. 1**. In fact, Biot number should be calculated for silver spherical probe as:

$$Bi = \frac{218785 \text{ W/m}^2\text{K}}{400 \text{ W/mK}} \times 0.01 \text{ m} = 5.47.$$

According to regular condition theory, the temperature difference between core and surface of silver probe is:

$$\frac{\bar{T}_{sf} - T_s}{\bar{T}_V - T_s} = \frac{1}{\sqrt{Bi_V^2 + 1.437Bi_V + 1}} = \frac{1}{\sqrt{3.57 + 1.437 \times 1.89 + 1}} = 0.467 \approx 0.5.$$

Here

$$Bi_V = 0.346Bi = 0.346 \times 5.47 = 1.89 \text{ or } \bar{T}_{sf} - T_s = 0.5 \times (\bar{T}_V - T_s) = 0.5\bar{T}_V - 0.5T_s.$$

It means that

$$\bar{T}_{sf} = 0.5T_s + 0.5\bar{T}_V = \frac{T_s + \bar{T}_V}{2}; \quad \frac{T_s + \bar{T}_V}{2} = 200; \quad \bar{T}_V \approx 300 \text{ } ^\circ\text{C}.$$

Temperature difference between the core and surface of silver probe is rather large.

The lumped-heat-capacity method is widely used in heat treating industry to evaluate heat transfer coefficients during quenching process [4–7]. It is used when the specimen temperature through its section is uniform. In this case the heat loss from the specimen is equal to the decrease in the thermal energy of the specimen which can be written as:

$$\alpha S(T_{sf} - T_m)dt = cpVdT \quad (6)$$

or

$$\alpha S(T_{sf} - T_m) = cpvV, \quad (7)$$

where  $v = dT / dt$  is cooling rate in  $^{\circ}\text{C/s}$ ;  $S$  is surface in  $\text{m}^2$ ;  $V$  is volume in  $\text{m}^3$ ;  $c$  is specific heat capacity of steel in  $\text{J/(kgK)}$ ;  $\rho$  is density of steel in  $\text{kg/m}^3$ .

Equation (7) can be derived from the general correlation of the regular condition theory [1] developed in 1954 which is true within  $0 \leq Bi_v \leq \infty$  and has a form:

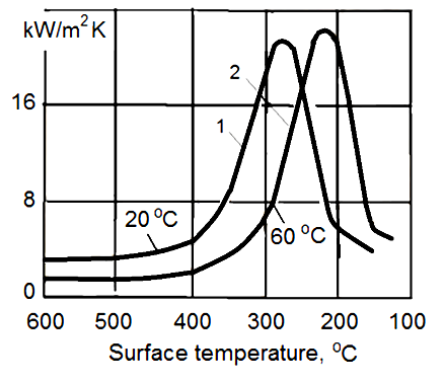
$$v = \frac{\alpha \psi S}{cpV} (T_{sf} - T_m). \quad (8)$$

When  $Bi \leq 0.25$ ,  $\psi \approx 1$  and then heat transfer coefficient is calculated as:

$$\alpha = \frac{cpvV}{S(T_{sf} - T_m)} \quad (9)$$

Eq. (9) follows from equations (7) and (8). In contrast to the lumped -heat- capacity method, Eq. (8) is applicable for smooth temperature distribution and also for maximal temperature gradients in the probes.

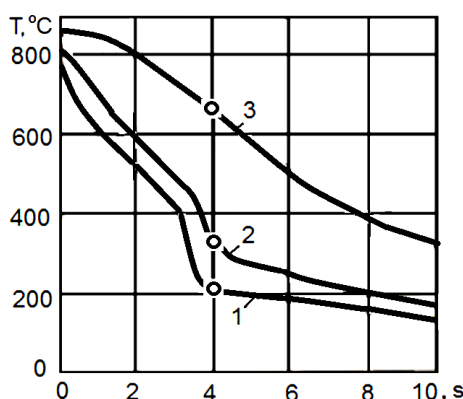
The lumped-heat-capacity method works perfectly in convection area with a temperature difference  $(T_{sf} - T_m)$  when  $Bi \leq 0.25$ . Such approach cannot be used for transient nucleate boiling processes where heat transfer coefficients reach  $200,000 \text{ W/m}^2\text{K}$  and more [11, 14]. A long ago, some incorrectness has been made by metallurgical engineers in FSU who used effective HTC to calculate  $Bi \leq 0.25$ . To show such incorrectness, author [2] used HTCs (**Fig. 1**), calculated by Eq. (9), for solving direct problem (temperature distribution vs. time in cylindrical sample of 20 mm in diameter) which is presented by **Fig. 2**.



**Fig. 1.** Heat transfer coefficients versus surface temperature of silver spherical probe 20 mm in diameter during quenching in water at different temperatures: first curve 1 is true for water at 20  $^{\circ}\text{C}$ ; the second curve 2 is true for water at 60  $^{\circ}\text{C}$  [15]

Using HTCs from **Fig. 1**, the temperature field was calculated in cylindrical specimen of 20 mm in diameter made of AISI 304 steel when quenching in water at 20  $^{\circ}\text{C}$  (**Fig. 1**). It turned out that surface temperature of cylindrical specimen during nucleate boiling maintains at the level of 200  $^{\circ}\text{C}$  instead of 100  $^{\circ}\text{C}$  [2]. It means that during testing of spherical silver probe 20 mm in diameter, a temperature difference between the surface and core of the silver probe was equal 100  $^{\circ}\text{C}$ . That is why error in temperature field calculation is 100 %.

Also, incorrect data are seen in area of film boiling processes. According to French (Table 3), during quenching in cold water and water salt (alkali) solutions film boiling is absent and a great temperature gradient through the section of probes takes place.



**Fig. 2.** Calculated temperature cooling curves for 20 mm cylindrical probe made of AISI 304 steel versus time when HTC was taken from Fig. 1: 1 is surface temperature; 2 is average volumer temperature; 3 is temperature at the center of cylindrical probe; o is transition time from film to nucleate boiling process

**Table 3**

Cooling time for surface temperature 500 °C and 150 °C in seconds of spherical steel probes cooled from 875 °C in 5 % NaOH water solution at 20 °C moving with 0.914 m/s, French [8]

Diameter, mm	500 °C	150 °C
6.35	0.043	0.69
12.7	0.058	0.60
25.4	0.055	0.82
65.5	0.065	0.59
120	0.09	0.95
180	0.10	1.15

As seen from Table 3, surface temperature of all probes drops from 875 °C to 150 °C within one second. According to Fig. 2, that happens between 10 and 20 seconds. It means that effective HTC cannot be used for temperature field calculation.

#### 4. Effective HTCs for core cooling time of steel parts calculations

Effective dimensionless numbers  $Kn$  and HTCs can be used for core cooling time and cooling rate calculations within the transient nucleate boiling process if they were evaluated as  $\tau_{nb} \equiv \tau_{conv}$ :

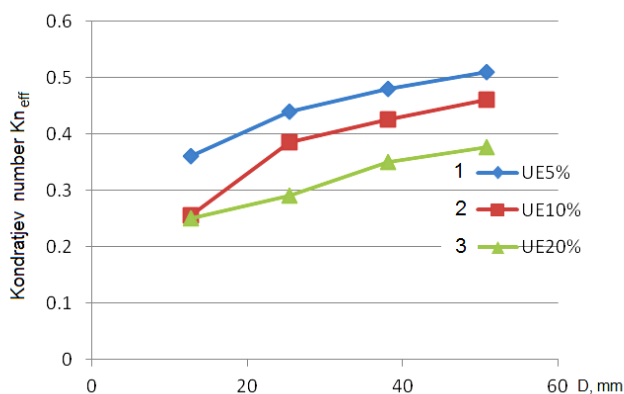
$$0.24k + 3.21 \ln \frac{\vartheta_I}{\vartheta_{II}} \equiv \left[ 0.24k + \ln \left( \frac{T_o - T_m}{T - T_m} \right) \right] \frac{1}{Kn_{eff}} \quad (10)$$

or

$$Kn_{ef} = \frac{\left[ 0.24k + \ln \left( \frac{T_o - T_m}{T - T_m} \right) \right]}{0.24k + 3.21 \ln \frac{\vartheta_I}{\vartheta_{II}}} \quad (11)$$

Here  $\tau_{nb}$  is duration of transient nucleate boiling process;  $\tau_{conv}$  is the same time calculated by equation which is true only in convection area;  $k=1, 2, 3$  for plate, cylinder, and spherical like forms;  $\vartheta_I$  is overheat at the beginning of nucleate boiling process;  $\vartheta_{II}$  is overheat at the end of nucleate boiling process.

Effective Kondrat'ev number allows calculating effective heat transfer coefficient for any configuration of steel part [1, 3]. Especially, it can be used for complicated configurations of steel parts covered by insulating polymeric layer. Some results of such approximate calculations are presented in **Fig. 3**.



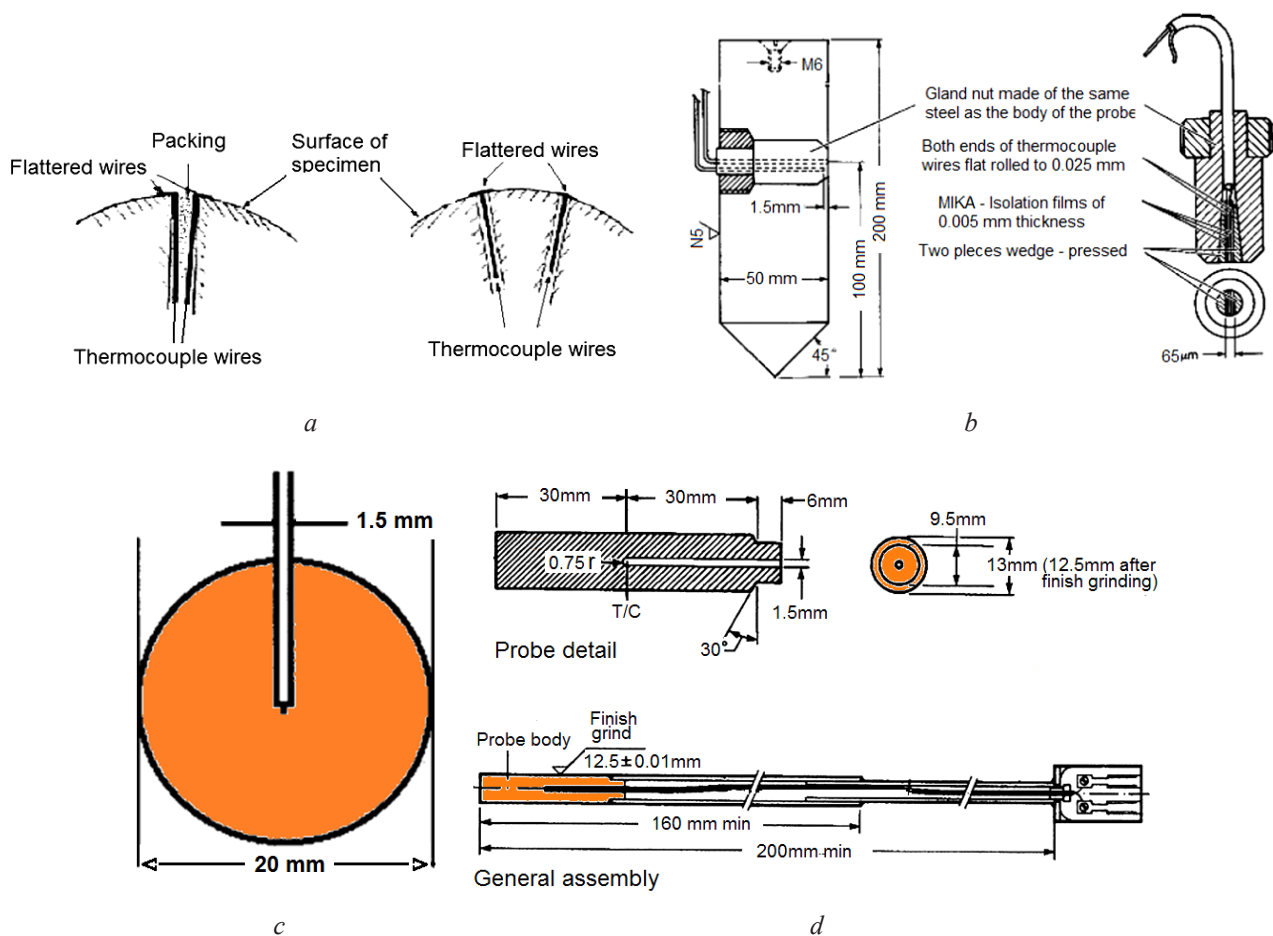
**Fig. 3.** Effective numbers  $Kn_{eff}$  vs. diameter of cylindrical samples made of AISI 4140 steel during their quenching in water PAG solutions of different concentration at 43 °C and agitation: 1 – UCON E of 5 % water solution agitated with 0.508 m/s; 2 – UCON E of 10 % water solution agitated with 0.25 m/s; 3 UCON E of 20 % water solution agitated with 0.508 m/s

Let's note that effective Kn numbers shown in **Fig. 3**, are true only for core cooling time calculation of steel parts and cannot be used for surface temperature calculation (**Fig. 2**).

### 5. Probes for testing cooling intensity of quenchants

To calculate correctly HTC, one should solve inverse problem (IP) based on accurate experimental data [16–19]. Inverse problem is very complicated task even for mathematicians because it is incorrect problem and mathematicians called it as ill – poised problem [20–23]. Only temperatures measured on the surface of probes or just below surface can provide correct data on HTCs after solving IP. The standard probes with one thermocouple instrumented at the core (center) are used for testing oils and polymers [24–26]. It is definitely not enough for obtaining data suitable for solving inverse problem when quenching standard probes in water salt solutions. Authors [9, 10] used several thermocouples instrumented in their probes for testing water and water solutions. Experimental data from testing such probes can be used for solving inverse problem. This fact is also understandable from the point of view of physics which states that single core cooling curve can be satisfied by hundreds variants of probe's surface cooling curves. It can be hundreds of local film boiling processes on the surface of probe which can perfectly produce the same core cooling curve. In the approach, presented by Eq. (10), is assumed that any film boiling is absent and only transient nucleate boiling and convection take place. That allows approximately calculate HTCs. Note that lumped – heat – capacity method doesn't work correctly for transient nucleate boiling processes due to huge thermal energy extraction by bubbles that creates temperature gradient even in small samples [27]. Proceeding from these facts, it is extremely important development of standard probes and their instrumentation by thermocouples suitable for any quenchant. Some developments in this field are presented by **Fig. 4**.

Accurate classical experimental data of French were received by testing numerous spherical probes of different diameters shown in **Fig. 4, a**. These very costly and painstaking experiments made by French in 1928–1930 are used by many scientists even today.



**Fig. 4.** Evolution of test samples since 1928 up to date: a are French samples with accurate thermocouples arrangement in their surfaces [8]; b is accurately designed Liscic/Nanmac probe (currently replaced by Liscic/Petrofer probe) to investigate quenching processes taking place on the surface of metal [9, 10]; c is silver standard probe 20 mm in diameter used in Former Soviet Union (FSU) [15]; d is standard Inconel 600 probe 12.5 mm in diameter currently used for testing different kinds of quenchants [24–26]

The most suitable probe for investigations transient nucleate boiling processes and some unusual phenomena taking place during quenching in liquid media was Liscic/Nanmac probe (**Fig. 4, b**) [9]. One tiny thermocouple was placed on the surface of the probe and another was instrumented 1.5 mm below its surface. Such accurately prepared probe was able investigate self-regulated thermal processes and other phenomena taking place during quenching. At present time Liscic/Nanmac probe is replaced by Liscic/Petrofer probe [10].

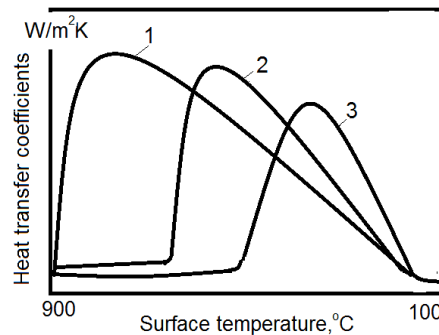
In Former Soviet Union (FSU) a silver spherical probe of 20 mm in diameter was widely used for testing liquid quenchants [15].

## 6. Discussion

It cannot be a universal correlation between heat flux density or HTC and surface temperature because crisis of heat transfer during boiling. If initial heat flux density is less than the first critical heat flux, film boiling is absent and HTC can be approximated by curve 1 (**Fig. 5**). If initial heat flux density is equal to critical heat flux than can be true curve 2. If initial heat flux density prevails critical heat flux, the developed full film boiling takes place and in this case is true curve 3. For these three conditions of cooling different curves, representing HTCs, are used as a function of



surface temperature. Moreover, according to authors [9] four modes of heat transfer during boiling can exist. And for each mode different curves should be used.



**Fig. 5.** Heat transfer coefficient (HTC) versus surface temperature during quenching in liquids:  
1 is HTC vs. surface temperature when initial heat flux density is less than critical heat flux;  
2 is HTC vs. surface temperature when initial heat flux density is equal to critical heat flux;  
3 is HTC vs. surface temperature when initial heat flux density prevails critical heat flux

So, there is no universal correlation between  $q$  and surface temperature of probe. The process of cooling is significantly simplified if any film boiling is absent. In this case only transient nucleate boiling and convection take place.

Computer simulation of quenching processes [28–31] require solving inverse problem [32–38] on the basis of testing probes with thermocouples instrumented on the surface of probe or close to surface.

As an advantage of this approach, concerning effective data, is a possibility of use Kondrat 'ev numbers  $Kn_{ef}$  within the transient nucleate boiling process to calculate core cooling time and core cooling rate of steel parts. It simplifies coupling nucleate boiling and convection modes and allows generalization already existing effective HTC's received by testing different probes with one thermocouple at the core.

The disadvantage of such approach is its restrictions:

- It can be used only for core cooling time and cooling rate calculation;
- It cannot be used for temperature field and stresses calculations during nucleate boiling process.

Further developments in this field could be careful investigation of transient nucleate boiling processes taking place during quenching of steel parts that require measurement of an overheat  $\Delta T = T_{sf} - T_s$  by thermocouples instrumented on the surface of probes. It is difficult to do because overheat is rather small that requires exact measurement of surface temperature (**Fig. 4 a, b**) combined with using sonar systems and video recording.

## 7. Conclusions

1. The lumped-heat-capacity method cannot be used correctly for HTC evaluation during transient nucleate boiling process because HTC at that moment is very large and it produces 100 °C temperature difference between surface and core of silver 20 mm spherical probe.

2. As a rule, the lumped-heat-capacity method provides effective HTC since it is calculated, during transient nucleate boiling process, as  $\alpha_{ef} = q / (T - T_m)$ .

3. Instead of the lumped-heat-capacity method a general cooling rate equation of regular thermal condition theory can be used for approximate calculation of the effective HTC.

4. The effective HTC is not a real value during nucleate boiling process and it can be used only for approximate core cooling rate and core cooling time calculations of steel parts during quenching in liquid media.

5. Theory and methods of solving inverse problem to evaluate HTC's are highly developed. However, accurate experimental data for this purpose are necessary. The most suitable



technique to provide experimental data for solving inverse problem are French and Liscic/Petrofer probes [37].

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