APPLICATION OF NONLINEAR MODELS FOR A WELL-DEFINED DESCRIPTION OF THE DYNAMICS OF ROTORS IN MAGNETIC BEARINGS

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Abstract

A research report has been submitted. It deals with implementing a method for a mathematical description of the nonlinear dynamics of rotors in magnetic bearings of different types (passive and active). The method is based on Lagrange-Maxwell differential equations in a form similar to that of Routh equations in mechanics. The mathematical models account for such nonlinearities as the nonlinear dependencies of magnetic forces on gaps in passive and active magnetic bearings and on currents in the windings of electromagnets; nonlinearities related to the inductances in coils; the geometric link between the electromagnets in one AMB and the link between all AMBs in one rotor, which results in relatedness of processes in orthogonal directions, and other factors. The suggested approach made it possible to detect and investigate different phenomena in nonlinear rotor dynamics. The method adequacy has been confirmed experimentally on a laboratory setup, which is a prototype of a complete combined magnetic-electromagnetic suspension in small-size rotor machinery. Different variants of linearizing the equations of motion have been considered. They provide for both linearization of restoring magnetic or electromagnetic forces in passive and active magnetic bearings, and exclusion of nonlinear motion equation terms. Calculation results for several linearization variants have been obtained. An appraisal of results identified the drawbacks of linearized mathematical models and allowed drawing a conclusion on the necessity of applying nonlinear models for a well-defined description of the dynamics of rotor systems with magnetic bearings.

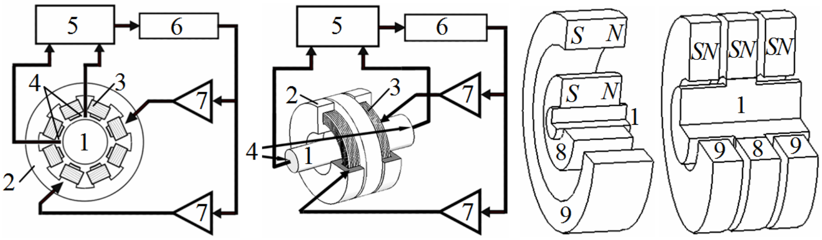
Keywords: rotor dynamics, magnetic bearings, mathematical model, linearization of equations of motion, nonlinear vibrations.

DOI: 10.21303/2461-4262.2016.00074

1. Introduction

A magnetic bearing (MB) is one of the variants of elastic-damping bearings. Its feature is the use of magnetic fields to provide stable rotor levitation. These fields create bearing force responses to rotor displacement in order to ensure automatic alignment of its bearing areas in the MB stator elements and a required level of bearing stiffness. MBs that are the most applicable from the practical viewpoint are active magnetic bearings (AMBs) [1–5] and passive magnetic bearings (PMBs) [6]. Some MB design options are shown in **Fig. 1**. It shows radial and axial AMBs with electromagnets (**Fig. 1, *a, b***) and a radial PMB with permanent annular magnets (**Fig. 1, *c, d***), and the following notations are introduced: 1 – rotor; 2 – stators; 3 – AMB windings; 4 – AMB position sensors; 5 – comparator in AMB control system; 6 – AMB control device; 7 – amplifiers feeding control voltages to AMB windings, which are formed according to the accepted control algorithm; 8 and 9 – movable and stationary annular permanent magnets.

Based on the possibilities of practical implementation of complete magnetic bearings of rotors, this study considers the options of using either radial or axial AMBs for stabilising a rotor over all five degrees of freedom or one AMB jointly with several PMBs in different design versions. The most practical approach would be to use a combination of magnetic bearings of different types in medium-sized high-speed rotor machinery, e. g., turbo-expanders, expander-generator, and expander-compressor units [4]. They can use two radial PMBs and one axial AMB arranged in the centre or at one end of the shaft. This is due to the design features such as the presence of one or two discs arranged on the rotor cantilevers [7].



a *b* *c* *d*

Fig. 1. MB design options: *a* – radial AMB; *b* – axial AMB; *c*, *d* – radial PMB

In considering the problem of describing the dynamics of rotors in different power machinery in which magnetic bearings are used as rotor bearing assemblies, a conclusion can be made on the need to develop special approaches to mathematical modelling with account for all specific features that this kind of elastic damping bearing introduces into rotor systems. Presently, there is a large variety of studies on this subject. However, they offer a simplified linear approach to mathematical modelling of rotor dynamics. However, rotor systems with nonlinear bearings, including magnetic ones, are known to exhibit nonlinear effects [8, 9]. Therefore, building refined mathematical models will enable increasing the accuracy of numerical computation of required dynamic parameters of rotors, magnetic bearings and control systems for active magnetic bearings. This will dramatically reduce the amount of experimental investigations and increase the effectiveness of research and development efforts.

2. Research objective and tasks

Paper [10] describes a method for interrelated modelling of nonlinear dynamics of rigid rotors in passive and active magnetic bearings. Its distinctive features are approach generality and completeness of accounting for the nonlinear interrelations of process occurring in such a system – electric, magnetic, and mechanical.

The objective of this study is a practical implementation of the method for substantiating the need to use nonlinear mathematical models for describing the dynamics of rotor systems with MB. In so doing, different methods of linearizing equations for streamlining mathematical models are discussed and analysed.

3. Mathematical model of rotor dynamics in a laboratory setup

The dynamic behaviour of a rotor in an MB was analysed for a laboratory setup of a rotor in a complete combined passive-active magnetic suspension, which was a prototype of the magnetic suspension for a rotor in an expander compressor unit (ECU). The setup is shown in **Fig. 2**. A schematic diagram of a complete magnetic suspension of a rotor, including two radial PMB1,2 and one axial AMB3, is shown in **Fig. 3**. Here, a PMB (**Fig. 1, *c***) is used as PMB1 and PMB2, and an AMB is used as AMB3 (**Fig. 1, *b***). A rigid rotor is considered because the vibrations caused by dynamic unbalance induce motion of the cylindrical and conical precession type. They are the most common ones in practice and are distinguished by excessive amplitudes, which make them especially dangerous.

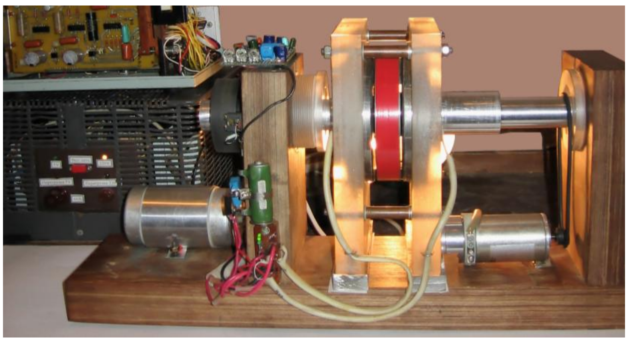


Fig. 2. Laboratory setup

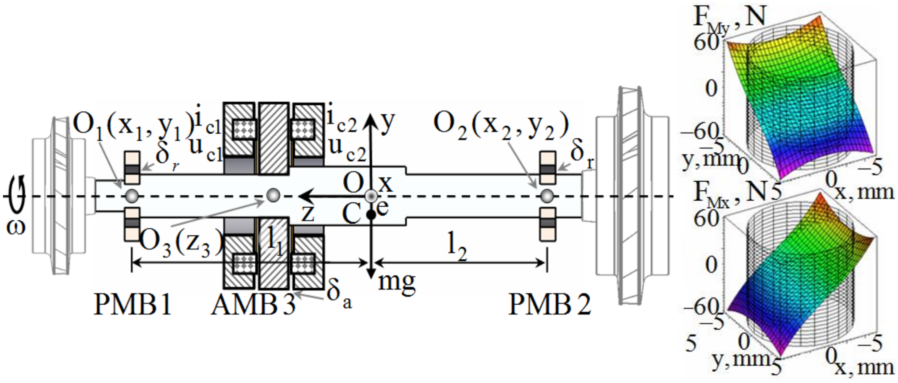


Fig. 3. Complete suspension for a laboratory setup rotor in an MB: *a* – schematic diagram; *b* – restoring magnetic forces in PMB1 and 2 vs. rotor displacements

If the energy of the AMB3 magnetic field is $W=W(x_1, y_1, x_2, y_2, z_3, \Psi_{c1}, \Psi_{c2})$, then the currents in the windings of its coils i_{c1} and i_{c2} are linked to total magnetic fluxes through the circuits of coils Ψ_{c1}, Ψ_{c2} (flux linkage in windings of respective AMB3 electromagnets) by the following expressions:

$$i_{c1}=\frac{\partial W(x_1,y_1,x_2,y_2,z_3,\Psi_{c1},\Psi_{c2})}{\partial \Psi_{c1}},\quad i_{c2}=\frac{\partial W(x_1,y_1,x_2,y_2,z_3,\Psi_{c1},\Psi_{c2})}{\partial \Psi_{c2}}. \tag{1}$$

We also assume that all generalised coordinates – displacements x_1,..., z_3, unbalance parameters e_1, e_2 and γ_1, γ_2, and gaps in the MB – δ_{r1}, δ_{r2}, and δ_a – have the same order of smallness. Then, with account for this assumption on smallness of generalised coordinates and their derivatives, the nonlinear addends of equations of motion can be considered small as compared to the linear terms. By excluding from consideration the addends of the equations of motion, with an order of smallness higher than three, we derive a completely coupled system of seven nonlinear differential equations describing the dynamics of this electromagnetic mechanical system:

$$\begin{cases} m_{11}\ddot{x}_1+m_{12}\ddot{x}_2+j(\dot{y}_1-\dot{y}_2)+f''_{x1}+f'''_{x1}+b_{x1}\dot{x}_1=-F_{Mx1}(x_1,y_1)+Q_{x1}+H_{x1}(t),\\ m_{22}\ddot{x}_2+m_{12}\ddot{x}_1-j(\dot{y}_1-\dot{y}_2)+f''_{x2}+f'''_{x2}+b_{x2}\dot{x}_2=-F_{My1}(x_1,y_1)+Q_{x2}+H_{x2}(t),\\ m_{11}\ddot{y}_1+m_{12}\ddot{y}_2-j(\dot{x}_1-\dot{x}_2)+f''_{y1}+f'''_{y1}+b_{y1}\dot{y}_1=-F_{Mx2}(x_2,y_2)+Q_{y1}+H_{y1}(t),\\ m_{22}\ddot{y}_2+m_{12}\ddot{y}_1+j(\dot{x}_1-\dot{x}_2)+f''_{y2}+f'''_{y2}+b_{y2}\dot{y}_2=-F_{My2}(x_2,y_2)+Q_{y2}+H_{y2}(t),\\ m\ddot{z}_3+f''_{z3}+f'''_{z3}+b_{z3}\dot{z}_3=-F_{Mz3}(x_1,y_1,z_1)-F_{Mz3}(x_2,y_2,z_2)-\dfrac{\partial W}{\partial z_3}+Q_{z_3}+H_{z_3}(t),\\ \dot{\Psi}_{c1}+r_{c1}\dfrac{\partial W}{\partial \Psi_{c1}}=u_{c1}(x_1,x_2,y_1,y_2,z_3);\qquad \dot{\Psi}_{c2}+r_{c2}\dfrac{\partial W}{\partial \Psi_{c2}}=u_{c2}(x_1,x_2,y_1,y_2,z_3), \end{cases} \tag{2}$$

where $f''_{qr}(x_1,...,z_3)$ and $f'''_{qr}(x_1,...,z_3)$ are nonlinear terms of the equations of motion due to inertia forces and the second and third-order potential field; $b_{x1,...,z3}$ are viscosity coefficients; $r_{c\,1,...,N}$ are active resistances in winding circuits; $u_{c\,1,...,N}$ are control voltages applied across AMB windings whose magnitude is formed according to the accepted control law depending on the rotor current position; m_{ks}, j are inertial and gyroscopic coefficients with the following values:

$$m_{11}=\frac{ml_2^2+J_1}{l^2},\ m_{12}=\frac{ml_1l_2-J_1}{l^2},\ m_{22}=\frac{ml_1^2+J_1}{l^2},\ j=\frac{\omega J_3}{l^2}. \tag{3}$$

Addenda $-F_{Mqr}$ are potential forces that depend only on the generalized coordinates. In this case, these are the magnetic forces in PMB1,2 (**Fig. 3, *b***). The magnetic forces dependencies were

obtained using the Maxwell tension tensor by solving a series of magnetic statics problems in the finite element statement for a fixed number of rotor magnet positions corresponding to certain discrete values of its displacement, though they can be described by analytical expressions as in [11]. Terms $-\partial W/\partial q_r$ are ponderomotive forces, i. e. the electromagnetic responses of AMB3. Their dependence on the generalised coordinates and currents in windings is suggested in this study to be obtained analytically by considering magnetic circuits with the use of equivalent circuits and the loop fluxes method [12]. Forces Q_{qr} are other generalized forces, in particular, the force of gravity, and $H_{qr}(t)$ are external time-dependent exciting forces and moments, in particular, caused by unbalance:

$$\begin{cases} H_{x1}(t) = m_{11}E_x + j\Gamma_x,\ H_{x2}(t) = m_{22}E_x - j\Gamma_x, \\ H_{y1}(t) = m_{11}E_y - j\Gamma_y, H_{y2}(t) = m_{22}E_y + j\Gamma_y, \\ E_x = e_1\cos(\omega t) - e_2\sin(\omega t),\ E_y = e_1\sin(\omega t) + e_2\cos(\omega t), \\ \Gamma_x = \gamma_1\sin(\omega t) + \gamma_2\cos(\omega t),\ \Gamma_y = \gamma_1\cos(\omega t) - \gamma_2\sin(\omega t). \end{cases} \tag{4}$$

Second-order nonlinear terms $f''_{qr}(x_1,...,z_3)$ and third-order nonlinear terms $f'''_{qr}(x_1,...,z_3)$ are not shown here due to their cumbersome notation; however, it is they that demonstrate the full co-relation between all generalized coordinates with the help of terms having no dependence on unbalance parameters.

A detailed technique for deriving system of equations (2) and all terms and expressions for magnetic energy for an axial AMB3, depending on flux linkages $\Psi=\{\Psi_{c1},\Psi_{c2}\}$ and generalized coordinates $q_r=\{x_1,y_1,x_2,y_2,z_3\}$, is described in [10].

4. Experimental research

Research was conducted for a rotor with a mass of 2.5 kg in a complete magnetic-electromagnetic suspension (**Fig. 2, 3**), in which, for AMB3, a unique control method and an algorithm were used, i. e. formation of $u_{c\,k}$ in (2) [13 and 14]: $u_{c2,1}=(u_{max}-2u_{min})z_3^2/(2\delta_a^2)\pm u_{max}z_3/(2\delta_a)+u_{min}$. The basic parameters have the following values: $l_1=0.118$ m; $l_2=0.166$ m; $J_1=0.00997$ kg×m²; $J_3=0.00347$ kg×m²; $\delta_r=5.5\times10^{-3}$ m; $\delta_r=3\times10^{-3}$ m; $e=6\times10^{-5}$ m; $\gamma=0.003$ rad; $u_{max}=24$ V, and $Q_{Rqr}=b_{qr}\times\partial q_r/\partial t$, where $b_{qr}=2.325$ kg/s. A laboratory setup with such parameters was developed as a prototype of a complete magnetic suspension for an ECU rotor. It was used for experimental studying of possible nonlinear dynamic phenomena in the system when the angular rotational speed changes within 0 to 3'000 rpm.

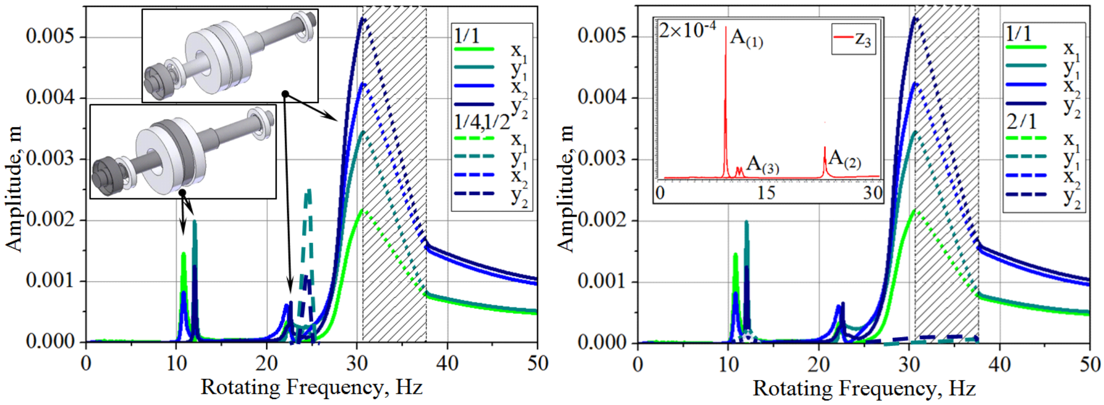
The result of a series of experiments was the amplitude-frequency response (AFR) shown in **Fig. 4**. It allows evaluating the presence of resonant modes in the area being investigated and the kind of rotor motion corresponding to different rotational speeds. Thus, the following was found:

– bifurcation of the first (~10.5 and ~12 Hz) and the second (~22.5 and ~31 Hz) resonances due to different PMB stiffness in the horizontal and vertical directions (anisotropy of bearings) due to different static equilibrium positions ($x_{1st}=x_{2st}=0$, y_{1st} and $y_{2st}\neq0$) with respect to centres of bearings that occur owing to the force of gravity;

– direct (~10.5 Hz) and reverse (~12 Hz) cylindrical precessions as well as direct (~22.5 Hz) and reverse (~36 Hz) conical precessions (**Fig. 4**, a shows vibration modes corresponding to these motions);

– loss of vibrations with transition from one stable mode to another stable mode (in **Fig. 4**, the crosshatched area within ~31–38 Hz is that of unstable motion, in which the vibration amplitudes, without introducing extra mechanical damping, exceed the PMB radial gap).

Besides, our analysis of the results detected in the system concerned the following: harmonic vibrations with an excitation (rotational) frequency, subharmonic and superharmonic vibrations, multiple sub and super resonances, and a link between radial and axial vibrations. A detailed description of the results is given in the following in comparison with the results of numerical modelling.



a *b*

Fig. 4. Experimental AFR response of the rotor and amplitude vs. rotational frequency response: *a* – subharmonics; *b* – superharmonics

5. Numerical research

During numerical modelling, the system of equations (2) was solved with the 4th-5th-order Runge-Kutta method for discrete angular speed values. The many-valuedness of the solution was checked and excluded by multiple computations for each frequency and different initial conditions. In doing so, stationary areas were searched for, whereas time intervals corresponding to transient processes were excluded from consideration. Hence, the results of the numerical analysis of forced vibrations are solutions for stationary areas and generalised coordinates x_1, y_1, x_2, and y_2 in the angular speed range of $0-100\pi$ rad/s. They are shown in **Fig. 5** as harmonics amplitudes A obtained by using the fast Fourier transform versus the driving force angular frequency ω_0 caused by the rotor's own unbalance. This frequency relates to the rotor angular speed as $\omega_0=\omega$.

The following notations are used in **Fig. 5**: $A_{(1)}$ is the first harmonic amplitude (**Fig. 5**), $A_{(1/n)}$ is the subharmonic amplitude (**Fig. 5, *a, c***) and $A_{(n)}$ is the superharmonic amplitude (**Fig. 5, *b, d***), where the number in parentheses is the multiplicity of the harmonic frequency with respect to the fundamental frequency ω_0, with the dashed lines showing the skeleton curves.

The natural frequencies of a nonrotating rotor that were calculated by using a linearized system of equations without account for damping [2π rad/s] are as follows: $p_{1x}=10.55$; $p_{1y}=11.90$; $p_{2x}=22.30$; $p_{2y}=30.90$. The curves in **Fig. 5** show the dynamic behaviour of a rotor in the investigated range and, in essence, they are the projections of 3-dimensional spectra on the coordinate planes OωA. Thereat, the dependence of the amplitude of the first harmonic of forced vibrations $A_{(1)}$ on the frequency of the harmonic driving force is the amplitude-frequency response, and the graphic representation of this dependence (**Fig. 5**) is the resonance curve. Analysis of these responses has shown that the first resonant mode (ω_{1z}) corresponds to axial vibrations.

Next, analysis of the results (**Fig. 5**) has demonstrated the following: superharmonic vibrations in the area of the second resonant mode (I); bifurcation of the second resonance due to anisotropy of PMB stiffness in the horizontal and vertical directions when at $\omega<\omega_{1x}$ and $\omega>\omega_{1y}$ the rotor's motion is of the direct cylindrical precession type, and in the range between these critical speeds $\omega_{1x}<\omega<\omega_{1y}$ the motion is of the reverse cylindrical precession type (II); super-resonant vibrations $\omega_{2x(2)}$, which coincide also with the inner resonance $\omega_{2x(2)}=\omega_{1y}$ (III); bifurcation of the third resonance due to anisotropy of PMB stiffness when at $\omega<\omega_{2x}$ and $\omega>\omega_{2y}$ rotor motion is the direct conical precession type, and in the range between these critical speeds ($\omega_{2x}<\omega<\omega_{2y}$) when the motion is the reverse conical precession type (IV); external resonance $\omega_{1x}+\omega_{1y}\approx\omega_0$ (V); subresonance vibrations $\omega_{1y(1/2)}$ amplified by inner resonance $\omega_{1y(1/2)}=2\omega_{1y}=2\omega_{2x(2)}$ (**Fig. 5**), with these subharmonic vibrations occurring at relatively big excitation frequencies and their amplitudes significantly exceeding the amplitudes of the first harmonic (VI); the form of resonance curves in the area of the third resonant mode (ω_{2x} and ω_{2y}) is specific to systems with rigid characteristics of the restoring force, which is true for PMB (VII); the third resonant mode is more dangerous than the second one because it is accompanied by a significant amplitude increase as during motion of the conical

precession type (angular vibrations) the flatness of gaps in the axial AMB is disturbed, resulting in a moment coinciding in the direction with the angular deflection of the rotor (VIII); in the area of frequencies, wherein two stable forced vibration modes with two different amplitudes are possible, a failure of vibrations is observed (IX); the fundamental and the superharmonic resonant vibrations in the axial direction are excited by a load acting in the radial direction (by the rotor's own unbalance), with the peaks of super-resonant axial vibrations coinciding with the peaks of the fundamental radial vibrations (**Fig. 5,** ***a*** – ***b***), which is the result of accounting for the interrelation between radial and axial generalised coordinates with nonlinear terms in the equations of motion (2) (X).

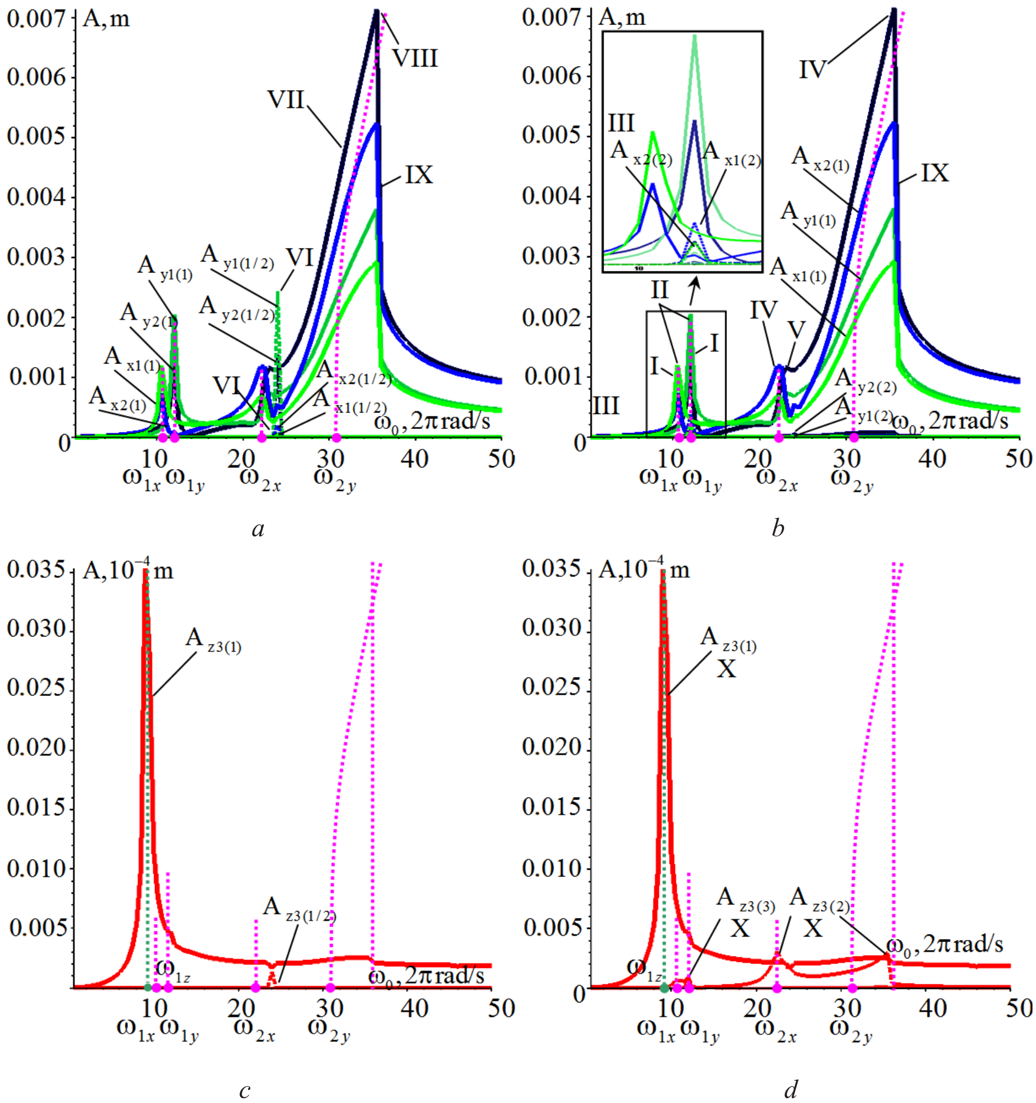


Fig. 5. Amplitudes of fundamental, sub- and superharmonics vs. driving force frequency:
a, *b* – x_1,y_1,x_2,y_2; *c*, *d* – z_3

The same resonant modes and phenomena were found in the system also during experimental research. The adequacy of the mathematical model representing a system of nonlinear completely mutually coupled by generalised mechanical coordinates x_1,...,z_3 and flux linkages Ψ_{c1} and Ψ_{c2} equations (with account for the control law, i. e. voltages $u_{1,2}$ also dependent on x_1,...,z_3) can be judged by the results of comparing the calculated data (**Fig. 5**) with experimentally obtained amplitude-frequency responses (**Fig. 4**) and dependencies of harmonic amplitudes that differ from the fundamental one by the driving force frequency. Thus, comparative analysis of the results has shown an identity for both qualitative representation of processes in the system and quantitative

determination of their parameters: for the amplitude, the difference is within 2–3 %, and for the values of resonant frequencies, the difference is within 0.2–0.5 %.

6. Methods of linearization of motion equations (dependencies of magnetic forces in bearings)

Fig. 3 shows the dependencies of restoring magnetic forces acting in a PMB on the rotor magnet during its displacement in the radial and axial directions. In this case, the radial forces are nonlinear and close to a cubic form. They can be approximated by third-degree polynomials depending on one variable.

Since the objective of the study is selecting the most adequate analytical model that mathematically describes the dynamic behaviour of rotors in magnetic bearings, comparative analysis was applied to results obtained using different models: #1) nonlinear – with account of nonlinear terms of inertia forces and nonlinear force characteristics of magnetic bearings (5); #2) nonlinear – without nonlinear terms of inertial forces, but with nonlinear MB force characteristics (5); #3) linearized – without nonlinear terms of inertial forces and with magnetic forces linearized with different methods, the forces obtained being based on equality of magnetic forces (6) or stiffnesses (7) in the static equilibrium position. Here, the expressions for magnetic forces have the form:

$$F_{M\rho}(\rho) = k_2\rho^3 + k_1\rho^2 + k_0\rho, \tag{5}$$

$$F_{M\rho 1}(\rho) = (k_2\rho_{st}^2 + k_1\rho_{st} + k_0)\rho, \tag{6}$$

$$F_{M\rho 2}(\rho) = \frac{\partial F_M(\rho_{st})}{\partial \rho}\rho = (3k_2\rho_{st}^2 + 2k_1\rho_{st} + k_0)\rho, \tag{7}$$

where ρ are radial displacements x or y, ρ_{st} are displacements corresponding to the static equilibrium positions x_{st} or y_{st}.

The dependence of the axial restoring magnetic force in the axial AMB3 is determined by the latter two equations for flux linkages (2) with account of the applied innovative control algorithm (i. e. formation of $u_{c\,k}$), and it is used in the fifth equation via term $-\partial W/\partial z_3$.

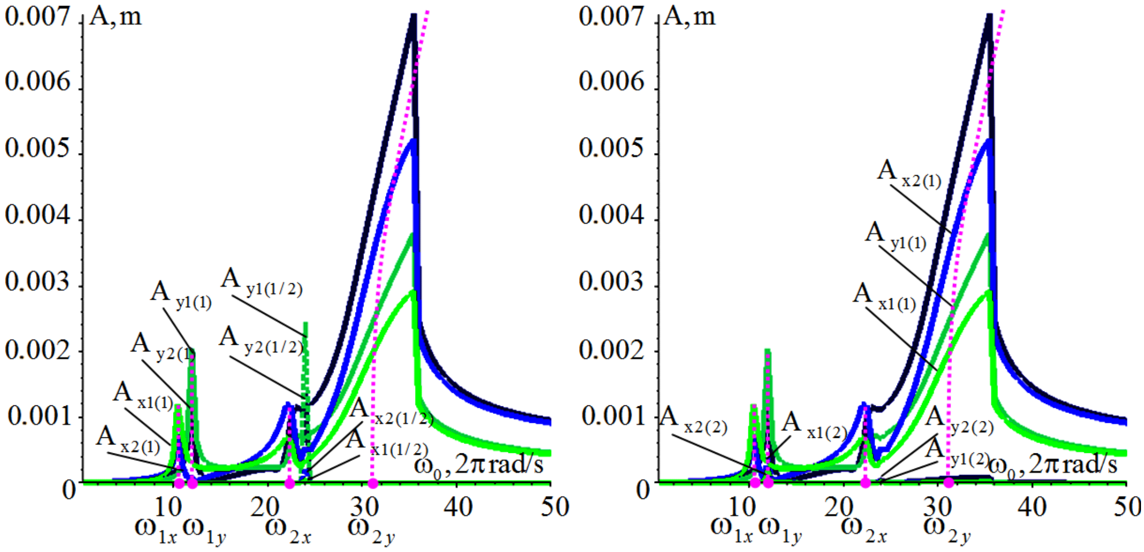
7. Comparative analysis of solution results obtained with linearized mathematical models

The practicality of applying nonlinear model #1 when investigating the dynamics of rotor systems with magnetic bearings was determined by comparing the above results (**Fig. 5**) with data obtained by using linearized analytical models.

Fig. 6 shows the results of calculations by using model #2. Their analysis shows that failing to account for nonlinear inertia force terms in equations (2) makes it impossible to describe vibrations excited in the axial direction, though this fact was established during both experimental studies (**Fig. 4**) and analytical experiments with model #1 (**Fig. 5**). This is indicative of a certain inadequacy of linearized model #2. The values of resonance frequencies shown in **Table 1** (for this and other variants of linearizing the analytical model) are entirely congruent, whereas the dependencies of the fundamental, super and subharmonics are completely identical to those obtained by using nonlinear model #1. This points to the possibility of using model #2, though with certain assumptions and reservations.

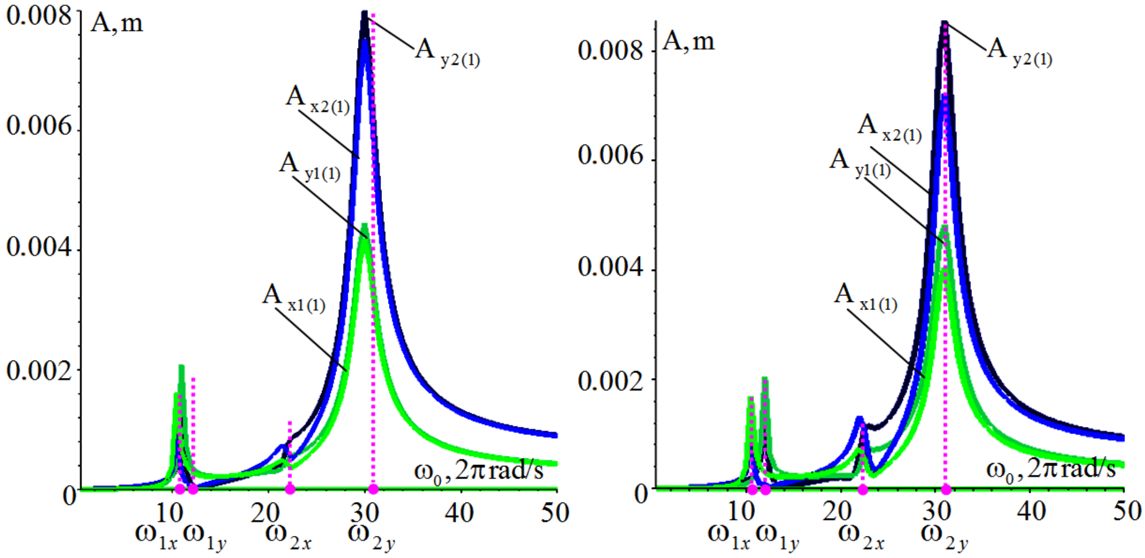
Fig. 7, a shows the results obtained with model #3 linearized by excluding nonlinear inertia force terms from equations (2) and using linear dependencies of magnetic forces (6).

This mathematical model has been found to be able to describe bifurcation of resonances in the system due to anisotropy of stiffness of radial PMB, and to describe the motion of the direct and reverse, and cylindrical and conical precession. However, the resonance modes frequencies are found with an error of ~6–8 % (**Table 1**), whereas the amplitudes corresponding to the resonance modes differ by ~10–30 % (**Fig. 7, *a***). Besides, the system behaviour is linear and super and subharmonics are absent; areas of frequencies with several stable forced vibration modes are not displayed; axial vibrations are not excited.



a *b*

Fig. 6. Harmonics amplitudes vs. driving force frequency (model #2) : *a* – fundamental harmonic and subharmonics; *b* – fundamental harmonic and superharmonics



a *b*

Fig. 7. Amplitude-frequency response: *a* – model #3 with forces $F_{M\rho 1}$; *b* – model #3 with forces $F_{M\rho 2}$

Table 1

Resonance frequencies calculated with different linearized models [2p rad/s]

Model #2 with forces $F_{M\rho}$	Model #3 with forces $F_{M\rho 1}$	Model #3 with forces $F_{M\rho 2}$
ω_{1x}=10.5	ω_{1x}=10.5	ω_{1x}=10.5
ω_{1y}=12.0	ω_{1y}=11.0	ω_{1y}=12.0
ω_{2x}=22.5	ω_{2x}=21.0	ω_{2x}=22.0
ω_{2y}=35.5 (break-down)	ω_{2y}=30.0	ω_{2y}=31.0
$\omega_{2x(2)}$=12.0; $\omega_{1y(1/2)}$=24.0	–	–

The situation with the description of dynamics phenomena appears to be better when models #3 with forces (7) are used. This is manifested by calculation results in **Fig. 7, *b*.**

In this case, the values of resonance frequencies are closer to those obtained with the nonlinear model and they are determined with an error of ~1–4 % (**Table 1**); however, their respective amplitudes also differ from reference ones by ~10–40 % (**Fig. 7, *b***).

Using this analytical linearized model, as in the previous case, demonstrates resonance bifurcation due to anisotropy of stiffness of radial PMBs, and the motion of the direct and reverse, and cylindrical and conical precession types. It is also impossible to describe any nonlinear effects of the super and subharmonic vibrations and resonances types, and the interrelation of vibrations in the axial and radial directions.

8. Discussion of the findings on the dynamics of a rotor in magnetic bearings

The result of this research, which is a follow-up of a large variety of studies, is the development and practical implementation of the method of mathematical description of linear and nonlinear rotor dynamics phenomena in systems with magnetic bearings of different types, which affect the vibration activity of power rotor machinery.

The advantage of the approach suggested is that the interrelationship of electric, magnetic and mechanical stationary and nonstationary processes can be accounted for as shown by the example of a laboratory setup.

Applying this approach to modelling the dynamics of rotor systems with magnetic bearings improves the dynamic parameters of a whole class of rotor machinery due to a more correct description of dynamic processes and phenomena occurring therein. In turn, this has the effect of cutting the cost of development activities at the design and commissioning stages, and reducing operational and power resources costs.

9. Conclusions

The developed imitation of the mathematical model built around the suggested mathematical modelling method has been used for numerical research into a complete magnetic suspension in a laboratory setup. The study has shown how the model can be used for investigating the mechanisms of excitation of spatial vibrations in rotating rigid rotors in an MB, finding out the conditions of existence of different resonant modes (including super, sub and inner resonances).

A variety of methods have been discussed, which allow linearizing the mathematical model suggested. It has been shown that each method has its drawbacks, and the linearized mathematical models based on them enable determining only the main dynamic characteristics with but a certain level of validity, thus constraining the limits of their applicability. It has been proved that nonlinear mathematical models should be used for refined analysis.

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