NONLINEAR METHODS OF CALCULATION OF STRUCTURES UNDER THEIR COMPLEX LOADING

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Abstract: Calculations of building structures, the development and improvement of calculation methods, the use of numerical methods in solving these problems are one of the promising areas.

The paper presents on the basis of the theory of plastic flow with hardening the equations of state in increments, the equations of virtual work, the geometric equations in increments for small elongations, shifts, and rotation angles.

The calculations take into account, depending on the intensity of the current load, the elastoplastic properties of the material, both the structure and the soil environment interacting with it, with a complex process of loading [1–3].

For numerical implementation, the equations arising from the conditions of Genius – applicable to concrete, Coulomb - Mohr – applicable to soil environments are applicable.

The information presented is implemented in the form of software systems that make it possible to obtain numerical results of solutions to problems in the calculation of underground structures.

An example of calculating a structure based on these methods using a software package is also given. The elastoplastic properties of the material, both the structure and the soil medium interacting with it, are taken into account during the complex process of their loading.

Keywords: soil medium, tensor, basic invariants, tunnel, software package, stresses, strains, bending moments.

1. Introduction

One of the directions for improving building structures is the development of calculation methods that take into account the actual properties of materials of building structures and soil. Due to contact with soil with rheological properties, the structure is a heterogeneous system.

Based on the theory and calculation equations used for the numerical implementation of the calculation, using the Plaxis computer software systems, the tunnel calculation problem is realized.

2. Methods

It is assumed that the structure and the soil environment interacting with it form a single connected system. The volume Q and surface q loads acting on it are divided into N_q increments:

$$Q = dQ^1 + dQ^2 + \cdots + \\ + dQ^j + \cdots + dQ^{N_q}.$$

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Under the influence of the indicated load, stresses σ and strains e occur in the system, which can be represented as:

$$\sigma = d\sigma^1 + d\sigma^2 + \cdots + \\ + d\sigma^j + \cdots + d\sigma^{N_q}.$$

$$e = de^1 + de^2 + \cdots + \\ + de^j + \cdots + de^{N_q}.$$

After acting on the system of the j-th load increment, the following stresses and strains appear in it:

$$\sigma^j = \sigma^{j-1} + d\sigma^j \qquad e^j = e^{j-i} + de^j,$$

$$\sigma^{j-1} = d\sigma^1 + d\sigma^2 + \cdots + d\sigma^{j-1},$$

$$e^{j-1} = de^1 + de^2 + \cdots + de^{j-1}.$$

The system consists of a continuous set of material particles, each of which is characterized by its position in a rectangular coordinate system $Ox_1x_2x_3$. Stresses and strains defined in each particle after the j-th increment of the load characterize the stress-strain state of the system.

Stresses and strains. The stress tensor is represented in the component form of the coordinate system $Ox_1x_2x_3$ as

$$T_\sigma = \sigma_{ks} i_k i_s, \quad k,s = 1,2,3;$$

$$[\sigma_{ks}] = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix},$$

$\sigma_{ks} = \sigma_{sk}$ if $k \neq s$, i_k are the unit vectors of the coordinate axes.

The first index σ_{ks} corresponds to a area with a normal parallel to the axis Ox_k, and the second to the direction of the axis Ox_s on which the stresses is projected.

A six-dimensional stretch Π_σ is introduced in which the coordinates of the point are equal to the tensor T_σ components. Three components σ_{ks}, $k \neq s$ are excluded due to symmetry $\sigma_{ks} = \sigma_{sk}$. Each tensor T_σ value in space Π_σ corresponds to a vector σ whose components form a column matrix

$$\sigma = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{bmatrix}.$$

The stress tensor T_σ can be represented as the sum of the stress tensor T_s and the spherical tensor T_{σ_0}. The following basic invariants of the stress tensor and stress deviator are introduced:

$$J_1(T_\sigma) = \sigma_{kk},$$

$$J_2(T_\sigma) = \frac{1}{2}\sigma_{ks}\sigma_{sk},$$

$$J_3(T_\sigma) = \frac{1}{3}\sigma_{ks}\sigma_{sm}\sigma_{mk},$$

$$J_1(T_s) = 0,$$

$$J_2(T_s) = \frac{1}{2}s_{ks}s_{sk},$$

$$J_3(T_s) = \frac{1}{3}s_{ks}s_{sm}s_{mk}.$$

Combining the basis invariants, it is possible to obtain a new system of three independent invariants [4, 5]:

$$\sigma_0 = \frac{1}{3}J_1(T_\sigma),$$

$$\sigma_i = \sqrt{J_2(T_s)},$$

$$\psi = \frac{1}{3}\arcsin\left(-\frac{3\sqrt{3}J_3(T_s)}{2(J_2(T_s))^{\frac{3}{2}}}\right),$$

$$-\frac{\pi}{6} < \psi < \frac{\pi}{6}.$$

By analogy with stretch Π_σ, the stretch Π_e of deformations is introduced and the *e* vector is defined as

$$e = [e_{11} \quad e_{22} \quad e_{33} \quad e_{12} \quad e_{23} \quad e_{31}]^{\mathrm{T}}.$$

In a similar way, the stress increment tensor, the strain increment tensor, and all subsequent definitions and equalities indicated above are introduced.

The principle of virtual work can be written in the form [3, 5]:

$$\int_\Omega \left[\left(\sigma_{ks}^{j-1} + d\sigma_{ks}^{j}\right)\delta e_{ks} - \left(Q_s^{j-1} + dQ_s^{j}\right)\delta u_s\right] d\Omega -$$
$$-\int_{S_q} \left(q_s^{j-1} + dq_s^{j}\right)\delta u_s dS = 0,$$

where δe_{ks} – virtual deformations; δu_s – virtual movements; Ω – volume occupied by the system; $S = S_u \cup S_q$ – surface bounding Ω; S_q – part of the surface on which the load is specified *q*; S_u – part of the surface on which displacements are specified $u\big|_{S_u} = 0(u_0)$, where u_0 – the vector of predetermined displacements.

Equations of state. Let's consider systems that under the influence of loads are able to deform, and their materials are endowed with the properties of elasticity and plasticity. The equations of state of these systems establish a relationship between stresses and strains or their increments. Together with static and kinematic, they form a system. By solving the system, it is possible to determine the stress-strain state in any particle of the structure or soil mass.

The components of the tensor of the increment of elastic deformations are related to the components of the tensor of the increment of stresses by the linear Hooke law

$$de_{ks}^{(e)} = C_{ksmn}^{(e)} d\sigma_{mn}, \tag{1}$$

the tensor of elastic coefficients $C_{ksmn}^{(e)}$ has the symmetry property. For isotropic media, dependence (1) has the form

$$de_{ks}^{(e)} = \frac{d\sigma_0}{3K}\delta_{ks} + \frac{1}{2G}ds_{ks},$$

where *K* – the bulk expansion modulus, *G* – the shear modulus.

The increments of plastic strains on the basis of the Mises maximum principle [5] are written in the vicinity of a regular point *f* of the loading function in the form

$$de_{ks}^{(e)} = d\lambda f_{\sigma_{ks}},$$

$$d\lambda = \text{const} > 0, \tag{2}$$

indicated

$$f_{,\sigma_{ks}} = \frac{\partial f}{\partial \sigma_{ks}}.$$

The loading function determines the loading surface

$$f\left(\sigma_{ks}, e_{ks}^{(p)}, \chi_m, k_m\right) = 0,$$

where χ_m – the hardening parameters and k_m – the mechanical constants of the deformable medium.

The loading surface may contain singular points at which its smooth components intersect. At these points, it is necessary to use the superposition principle proposed by Koyter, according to which instead of (2) one should use a relation

$$de_{ks}^{(p)} = \sum_{n=1}^{N} d\lambda_n f_{\sigma_{ks}}^{n},$$

where *N* is the number of smooth component surfaces that define the loading surface and intersect at a given singular point.

If the loading function depends only on stresses and does not depend on the history of loading, then in this case the loading surface is called the yield surface and its equation is written in the $f(\sigma_{ks}) = 0$.

In the process of deformation, this surface does not change, i.e. is fixed in the Π_σ stress space, and can have singular points.

Load functions. The equations of state depend on the derivatives of the loading function. Consequently, the loading functions specify models of deformable media. As experimental data and field observations have shown, media with different properties are described by various loading functions. So, for soil and mountain environments working under conditions of plane deformation, the Coulomb-Mohr condition [6, 7] can be used as a loading function, for Coulomb-Mohr condition for concrete and reinforced concrete [7–9] or the Geniev condition [7].

3. Results

Tunnel calculation. To solve the calculation problem of calculating the tunnel, the Plaxis 2010 software package is used.

The design scheme of the system consists of a cylindrical tunnel, a three-layer soil medium and the foundation of the structure located on the surface of the soil massif (**Fig. 1**).

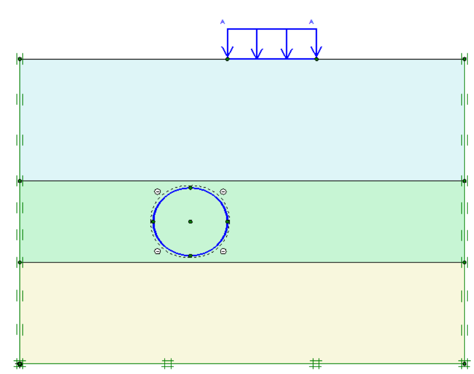

Fig. 1. The design scheme of the system

To carry out the calculation, a volume of 1 m wide, 60 m long and 45 m high is cut out in a semi-infinite soil mass. Boundary conditions are imposed on its sides, representing pinching with free vertical displacement. The sole of the array is rigidly pinched. The origin is located in its lower left corner. The *x* axis is directed to the right, and the *y* axis is up. The diameter of the tunnel is 10 m, and its axis has coordinates x_0=25 m, y_0=20 m. On the upper edge of the soil mass is the foundation of the structure, having a length of 12 m, on which a uniformly distributed vertical load with an intensity of –100 kN/m^2 acts. The distance to its left edge is 28 m. Groundwater is located at a depth of 3 m from the upper edge of the massif.

The topsoil consists of sandy loam and has the following mechanical characteristics: dry weight 16 kN/m^3, horizontal and vertical permeability 0.2 m/day, Young's modulus 60000 kN/m^2, Poisson's ratio 0.3, adhesion 2 kN/m^2, internal friction angle 27º. The middle layer consists of sand with mechanical characteristics: dry weight 17 kN/m ^{3}, horizontal and vertical permeability 1 m/day, Young's modulus 80000 kN/m^2, Poisson's ratio 0.3, adhesion 1 kN/m^2, internal friction angle 31º. The lower layer consists of from clay with mechanical characteristics: dry weight 16 kN/m^3, horizontal and vertical permeability of 0.01 m/day, Young's 10000 kN/m^2 modulus, Poisson's ratio is 0.4, clutch 5 kN/m^2, the internal friction angle of 26º.

The mechanical characteristics of the base plate material are equal: tensile stiffness, compression 10000000 kN/m, bending stiffness 1600000 kN/m^2, Poisson's ratio 0.2. The mechanical characteristics of the material of the tunnel ring are equal to: compressive tensile rigidity of 16000000 kN/m, bending stiffness of 16300000 kN/m^2, Poisson's ratio of 0.2. Discretization of the initial equations and discretization of the system is performed by the finite element method, which were taken as triangles with 15 nodal points. For the initial conditions of the soil massif: the displacement of any particle equal to zero, and the initial stress state is caused only by pressure from the own weight of the soil and pressure from groundwater. The influence of the weight of the tunnel ring and the weight of the foundation of the structure with the load acting on it were not taken into account.

The greatest compressive stresses are located at the base of the array and are respectively 455.62 kN/m^2 and 398.75 kN/m^2. The resulting displacements from these effects are reset to zero.

Further calculation is carried out in *two stages*. *On the first* of them, the stress-strain state of the system is determined only from the action of the foundation of the structure and the load applied to it. The construction is erected before the construction of the tunnel.

The greatest displacements occur in the soil under the right side of the foundation of the structure, they are 3.06 cm. The greatest compressive stresses are located at the bottom of the soil massif, they are 213.6 kN/m^2.

At the second stage of the calculation, the stress-strain state of the system is determined after the completion of the tunnel construction.

The greatest displacement, equal to 2.44 cm, is located under the foundation of the structure. The tunnel ring did not receive displacements. This is because the weight of the ring has increased, but the weight of the soil removed from the tunnel has decreased. The highest stresses are located in the ground near the ring of the tunnel and on the sole of the soil mass. They are 210 kN/m^2. Their largest values of shear stresses have both positive and negative values equal to 66 kN/m^2. They are located in the ground, both under the foundation and on the sides of the tunnel ring. The greatest compressive axial forces are 1290 kN/m, and the transverse ones are 112 kN/m. The largest axial forces are an order of magnitude larger than the transverse ones. The greatest value of bending moments in the tunnel ring is 249.6 kN/m.

Displacements of the lower left edge of the foundation of the structure depending on the number of iterations (there are 33). The displacements after the first calculation stage are 2.9 cm, then they decrease to 2.23 cm at the second calculation stage. This is caused by a change in the system stiffness due to the construction of the tunnel.

4. Discussion and conclusions

Based on the use of the theory of plastic flow with hardening, based on the application of the Mises maximum principle, the following equations are obtained in a form convenient for application to the calculation of structures:

– equations of state in increments; these equations are applicable both for structures and for the soil environment, which form a single joint system;

– equation of virtual work, which is convenient for discretization of the system;

– geometric equations in increments at small elongations, shifts, and rotation angles.

In an invariant form, which is convenient for numerical implementation, the equations of the loading functions are derived from the conditions:

– Geniev, which are applicable for concrete and reinforced concrete; from them in a particular case follows the Mises condition used for metals and alloys;

– Coulomb-Mohr, which are applicable for soil environments; of them in the particular case follows the Tresca-Saint-Venant condition, which is also used for metals.

A modification of the loading functions is presented, which allows one to take into account volumetric plastic deformations not only from the shape of the medium, but also from comprehensive tension and compression.

Coulomb-Mohr loading surface has singular lines and points. Their features are investigated and some recommendations for their elimination are given.

The theory stated above is implemented in the form of software systems [2, 9] and Plaxis, which is used to obtain numerical results for solving the tunnel calculation problem. The calculation is performed in two stages and makes it possible to analyze the stress-strain state of the system.

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Application of the indicated methods to the calculations of structures, taking into account the modification of the loading functions, allowing to take into account volumetric plastic deformations not only from the shape of the medium, but also from comprehensive tension and compression. Implementation in the form of software systems will increase the accuracy of calculations of the tasks to be solved.

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