1. Introduction

The concept of bifurcations of stationary states is widely used in various fields of science. Interest that manifests itself to them recently in many countries is explained by the fact that the task of identifying the transformation of the qualitative changes into qualitative changes is key in revealing the fundamental secrets of nature and the underlying processes and phenomena occurring in it.

The phenomenon of bifurcation (Latin Bifurcus – bifurcation, branching) in the phase space or in the space of states of the dynamic system corresponds to a qualitative change in the character of the motion of the output mechanical system.

Mathematical pendulums with an arbitrary number of links are constructed in [9]. An analysis of the dynamics development of pendulum systems is given in [10]. The research results of bifurcations of equilibrium states of an inverted double pendulum loaded at the upper end by an asymmetric follower force are presented in [1–10]. The equilibrium states of a single inverted pendulum are considered in this paper.

2. Methods

The design scheme of the inverted single pendulum is shown in Fig. 1, consists of a weightless rod OA1 of length l1 and a material point A1 of mass m1. At the point of support O there is visco-elastic (rigid, soft or linear) element of support O there is visco-elastic (rigid, soft or linear) element (realized, for example, by means of a spiral spring and a hydraulic damper). Let \( c_i \) be the stiffness of the spring, \( \mu_i \) – the viscosity coefficient in the hinge \( O \), and also takes into account the effect of external friction. The upper end of the pendulum is resiliently fixed by means of a horizontal spring of rigidity \( c \). Similarly to [2], let’s assume that in the vertical position of the pendulum all elastic elements are deformed. The follower force \( P \) is also applied to the upper end of the pendulum.

The equations of the perturbed motion of the pendulum have the form:

\[
m_1 \dot{\psi}_1 = m_1 g l_1 \sin \phi_1 + P \sin([1 - k] \phi_1 - \delta) + e \cos([1 - k] \phi_1 - \delta) - \left( q_1 Q'_1 + q_2 Q''_1 + q_3 Q'''_1 \right) \cos \phi_1 - (q_4 M'_1 + q_5 M''_1 + q_6 M'''_1).
\]

where

\[
m_1 \dot{\psi}_1 = m_1 g l_1 \sin \phi_1 + P \sin([1 - k] \phi_1 - \delta) + e \cos([1 - k] \phi_1 - \delta) - \left( q_1 Q'_1 + q_2 Q''_1 + q_3 Q'''_1 \right) \cos \phi_1 - (q_4 M'_1 + q_5 M''_1 + q_6 M'''_1).
\]

\[\phi_1 + (q_{11} + q_{12} + q_{13}) P \cos \phi_1 = \frac{2\pi}{\pi} \sin \phi_1 + P \sin \left( (1 - k) \phi_1 - \delta \right) + \cos \left( (1 - k) \phi_1 - \delta \right),\]

Equations (2) at

\( \delta = 0 \) and \( e = 0 \)

admit a solution \( \phi_1 = 0 \), which corresponds to the vertical position of equilibrium of the pendulum. Let’s consider the problem of the existence of equilibrium positions of the pendulum for

\[\delta \neq 0 \) and \( e \neq 0.\]

\[f_1(\psi_1, \delta, \varepsilon) = 0,\]

\[1\]
\( f_1(\varphi_0, \delta, \tau) = \frac{2\pi}{\tau} \sin \varphi_0 + \frac{\varphi_0}{\pi} \sin\left[(1 - k)\varphi_0 - \delta\right] + \frac{\varphi_0}{\tau} \cos\left[(1 - k)\varphi_0 - \delta\right] - \frac{2\pi}{\tau} \frac{\tau_1}{\tau_2} \frac{\varphi_0}{\tau_1} + \frac{\varphi_0}{\tau_2} \frac{\varphi_0}{\tau_1} \cos \varphi_0 - \varphi_0 \cos \varphi_0 - \varphi_0 \sin \varphi_0 \right) \)

(4)

It is logical to assume that there exists a non-vertical state of equilibrium that can be found by direct integration of the differential equation of motion of the pendulum, solved the corresponding Cauchy problem for fixed values of the parameters.

To do this, we use the parameter extension method developed by the Japanese scientist Y. A. Shinohara.

3. Results

The results of the solution of the Cauchy problem showed that only in the case of springs with soft characteristics \( q_2 = q_{12} = 1 \) and \( q_1 = q_{11} = q = q_{12} = 0 \) in equation (2) bifurcations of pendulum equilibrium states occur. In the cases of rigid \( (q_1 = q_{11} = 1) \) and \( q_2 = q_{12} = q = q_{13} = 0 \) and linear \( (q_1 = q_{11} = 1 = q_{12} = q = q_{13} = 0) \) characteristics of the elastic elements of the pendulum, the dependences are single-valued.

Unique non-vertical (for \( \tau \neq 0 \)) equilibrium position of the pendulum is corresponded to each value of the linear eccentricity of the follower force. In this case, the larger the value \( \varphi_0 \), the greater the angle of pendulum deviation \( \varphi_0 \) from the vertical.

A comparative analysis of the pendulum's state shows that for linear characteristics this angle is larger than for stiff characteristics, consistent with intuitive considerations. By varying the parameter \( b_0 \), characterizing the helical spring in the hinge O, it is found that increasing or decreasing \( b_0 \) leads to a change in the configuration of the equilibrium state curve. For sufficiently small values of \( \tau \) for \( b_0 = 0.05 \), there are three equilibrium states of the pendulum, for \( b_0 = 0.25 \) there are five of them, and for \( b_0 = 0.5 \) only one \( (q_1, q_0) = 0 \).

The parameter \( b_0 \) characterizing the horizontal spring at the upper end of the pendulum does not affect the configuration of the equilibrium state curve.

For a pendulum with rigid characteristics of elastic elements, a change in the orientation parameter of the follower force leads to the fact that for small angular eccentricity the pendulum has only one non-vertical equilibrium state. The effect of this eccentricity is visible only for values of \( \delta \) close to \( \pi \).

For a pendulum with soft characteristics of elastic elements, a similar effect of the follower force orientation parameter on the equilibrium curves is observed.

4. Discussions of results

The theoretical and practical significance of the results is the development of analytical-numerical methods for constructing the dependences of the equilibrium values of the generalized coordinates of single and double inverted mathematical pendulums on the parameters of the linear and angular eccentricities of the follower force, as well as its orientation parameter. The obtained results for different types of characteristics of elastic elements deepen the scientific base of design engineers on the influence of the parameters of pendulum systems on their dynamic behavior and can be used in research and design organizations in the modeling of hinges of control surfaces of aircrafts, railway trackers, in calculations of dynamic vibration dampers of building structures, in predicting the dynamic behavior of one-dimensional pipelines, in the mechanics of the number of crews, while predicting the functional capabilities of machine elements and mechanisms with pendulum systems.

References